Universidad Católica del Norte, Economic Department

Spatial Econometrics: Exam 1

Professor: Mauricio Sarrias

November 5, 2015

1. Consider the following Spatial Lag Model:

$$\mathbf{y} = \rho \mathbf{W} \mathbf{y} + \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \tag{1}$$

   a) Is this model inconsistent if it is estimated by OLS? Assume what you think is necessary to assume. Show all your work.
   b) Is this model homokedastic? Assume what you think is necessary to assume. Show all your work.
   c) Find the marginal effects for this model. Assume what you think is necessary to assume. Show all your work.
   d) Assume further that $\boldsymbol{\varepsilon} \sim \mathrm{N}(\mathbf{0}_n, \sigma^2 \mathbf{I}_n)$. Find the log-likelihood function for this model and the ML estimates. Show all your work.
   e) According to your answer in (d), outline the steps of some algorithm to estimate the model by ML.

2. What is the main contribution of Ord (1975)? Explain in detail.

3. Following Lesage's book, explain in detail the "time-dependence motivation" of spatial models. Show all your work.

4. Explain the Monte Carlo Moran's I test and the logic behind it in detail. Show all your work.

Some important results are the followings:

$$\frac{\partial(\rho\mathbf{W})}{\partial\rho} = \mathbf{W} \tag{2}$$

$$\begin{aligned}\frac{\partial\mathbf{A}}{\partial\rho} &= \frac{\partial(\mathbf{I}-\rho\mathbf{W})}{\partial\rho} \\ &= \frac{\partial\mathbf{I}}{\partial\rho} - \frac{\partial\rho\mathbf{W}}{\rho} \\ &= -\mathbf{W}\end{aligned} \tag{3}$$

$$\frac{\partial\log|\mathbf{A}|}{\partial\rho} = \mathrm{tr}(\mathbf{A}^{-1}\partial\mathbf{A}/\partial\rho) = \mathrm{tr}\left[\mathbf{A}^{-1}(-\mathbf{W})\right] \tag{4}$$

Let $\boldsymbol{\varepsilon} = \mathbf{A}\mathbf{y} - \mathbf{X}\boldsymbol{\beta}$, then:

$$\frac{\partial\boldsymbol{\varepsilon}}{\partial\rho} = \frac{\partial(\mathbf{A}\mathbf{y}-\mathbf{X}\boldsymbol{\beta})}{\partial\rho} = -\mathbf{W}\mathbf{y} \tag{5}$$

$$\frac{\partial\boldsymbol{\varepsilon}^\top\boldsymbol{\varepsilon}}{\partial\rho} = \boldsymbol{\varepsilon}^\top(\partial\boldsymbol{\varepsilon}/\partial\rho) + (\partial\boldsymbol{\varepsilon}^\top/\partial\rho)\boldsymbol{\varepsilon} = 2\boldsymbol{\varepsilon}^\top(\partial\boldsymbol{\varepsilon}/\partial\rho) = 2\boldsymbol{\varepsilon}^\top(-\mathbf{W})\mathbf{y} \tag{6}$$

$$\frac{\partial\mathbf{A}^{-1}}{\partial\rho} = -\mathbf{A}^{-1}(\partial\mathbf{A}/\partial\rho)\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{W}\mathbf{A}^{-1} \tag{7}$$

$$\frac{\partial\,\mathrm{tr}\left(\mathbf{A}^{-1}\mathbf{W}\right)}{\partial\rho} = \mathrm{tr}\left(\partial\mathbf{A}^{-1}\mathbf{W}/\partial\rho\right) \tag{8}$$